2017-2018

Master φCS

SOFT MATTER

PRACTICES 2

Surface energy: macroscopic description

1 Jurin's law

When a narrow, cylindrical, glass tube of radius R is dipped vertically into a "wetting" liquid of density ρ, the liquid level within the tube rises a height H above the free surface as a consequence of surface tension (see Fig. 1). Jurin's law relates the height of liquid H to the radius R of the tube.

1. *capillary* tube: we assume that $R \ll \ell_c = \sqrt{\gamma/(\rho g)}$. Show that the liquid-air interface in the tube can be approximated as a spherical cap.
2. Derive Jurin's law using three different methods:
   - using hydrostatic law and Laplace's law;
   - using minimization of potential energy;
   - using balance of forces.

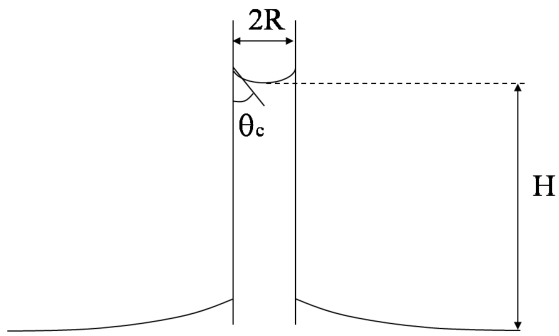

Figure 1: Rise of liquid in a capillary tube

2 Capillary adhesion

A drop of water forms a *capillary bridge* between two parallel glass plates. Show that, when gravity action is neglected, the plates experience an attractive force along the normal direction, with amplitude:

$$F = \gamma\pi R\,(2\sin\theta + 2\cos\theta R/H - 1)\,,$$

where H is the distance between the plates, R the radius of the axisymmetric bridge (taken at the mid-distance between the plates), and γ the water(-air) surface tension.
Which is the dominant term in the limit $H \ll R$?

3 Plateau-Rayleigh instability

A cylindrical column of liquid (e.g. water that springs from garden hose) spontaneously breaks in spherical drops in order to reduce its total surface area. This instability is known as "Plateau-Rayleigh instability". We want to estimate the minimal wavelength of perturbations that can induce a destabilization of the cylindrical geometry.

3.1 Rough estimation

1. Compare the lateral surface area of (1) a cylinder of volume V and radius R_c^0, with the total surface area of (2) N spherical drops of radius R_s and having same total volume, and determine the minimal radius of the drops for which the surface energy of configuration (2) is smaller than configuration (1).
2. Deduce a rough estimation of the minimal wavelength of a perturbation that destabilizes the cylindrical column of liquid.

3.2 "Rigorous" treatment

We consider the following axisymmetric mode of perturbation of the cylindrical geometry: the radius of the surface at abscissa x is given by:

$$r_c(x) = R_c + \delta r_c \cos(k.x),$$

with $k = 2\pi/\lambda$.

1. Using volume conservation over one wavelength, show that R_c is related to the unperturbed radius R_c^0 of the cylinder through:

$$R_c = \left({R_c^0}^2 - (\delta r_c)^2/2\right)^{1/2}.$$

2. Simplify this relation for small perturbations ($\delta r_c \ll R_c^0$).
3. Assuming again small perturbation amplitude, show that the variation of energy (per unit wavelength) between the perturbed and unperturbed geometries is

$$\Delta E = 2\pi\gamma \frac{\lambda}{R_c^0} \frac{(\delta r_c)^2}{4} \left({R_c^0}^2 k^2 - 1\right),$$

   where γ is the surface tension.
4. Conclude that destabilizing modes satisfy $\lambda > 2\pi R_c^0$.